Deep Poisson Factor Modeling

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Contributions

- A deep architecture for topic models based entirely on Poisson Factor Analysis (PFA) modules.
- Inherent shrinkage in all layers, thanks to the DP-like formulation of PFA.
- Block updates for binary units improve mixing.
- PFA modules can be used to easily build discriminative topic models.
- Efficient MCMC inference scales as function of the number of *non-zeros* in data and binary units.
- Scalable Bayesian inference algorithm based on Stochastic Variational Inference (SVI).

Poisson factor analysis as a module

Assume $\mathbf{x}_n$ is an M-dimensional vector containing word counts for the n-th of N documents, where M is the vocabulary size. We impose the model

$$\mathbf{x}_n \sim \text{Poisson}\left(\boldsymbol{\Psi}(\boldsymbol{\theta}_n \circ \mathbf{h}_n)\right) , \qquad (1)$$

where

- $\boldsymbol{\Psi} \in \mathbb{R}_+^{M\times K}$, factor loadings matrix with K factors.
- $\boldsymbol{\theta}_n \in \mathbb{R}_+^{K}$, factor intensities.
- $\mathbf{h}_n \in \{0,1\}^K$, binary units indicating which factors are active for observation n.
- Symbol $\circ$ denotes element-wise (Hadamard) product.

Prior specification [2]:

$$\begin{aligned} x_{mn} &= \textstyle\sum_{k=1}^K x_{mkn} , & x_{mkn} &\sim \text{Poisson}(\lambda_{mkn}) , & \lambda_{mkn} &= \psi_{mk}\theta_{kn}h_{kn} , \\ \boldsymbol{\psi}_k &\sim \text{Dirichlet}(\eta \mathbf{1}_M) , & \theta_{kn} &\sim \text{Gamma}(r_k,(1-b)b^{-1}) , & h_{kn} &\sim \text{Bernoulli}(\pi_{kn}) . \end{aligned} \qquad (2)$$

Note that η controls for the sparsity of $\boldsymbol{\Psi}$, while r_k accommodates for over-dispersion in $\mathbf{x}_n$ via $\boldsymbol{\theta}_n$.

PFA module: Conditioned on $\mathbf{h}_n$, we express

$$\mathbf{x}_n \sim \text{PFA}(\boldsymbol{\Psi}, \boldsymbol{\theta}_n, \mathbf{h}_n; \eta, r_k, b) . \qquad (3)$$

Deep representations with PFA modules

Develop a deep prior specification for $\mathbf{h}_n$ as

$$\begin{aligned} \mathbf{x}_n &\sim \text{PFA}\left(\boldsymbol{\Psi}^{(1)}, \boldsymbol{\theta}_n^{(1)}, \mathbf{h}_n^{(1)}; \eta^{(1)}, r_k^{(1)}, b^{(1)}\right) , & \mathbf{h}_n^{(1)} &= \mathbf{1}\left(\mathbf{z}_n^{(2)}\right) , \\ \mathbf{z}_n^{(2)} &\sim \text{PFA}\left(\boldsymbol{\Psi}^{(2)}, \boldsymbol{\theta}_n^{(2)}, \mathbf{h}_n^{(2)}; \eta^{(2)}, r_k^{(2)}, b^{(2)}\right) , & &\vdots \\ &\vdots & \mathbf{h}_n^{(L-1)} &= \mathbf{1}\left(\mathbf{z}_n^{(L)}\right) , \\ \mathbf{z}_n^{(L)} &\sim \text{PFA}\left(\boldsymbol{\Psi}^{(L)}, \boldsymbol{\theta}_n^{(L)}, \mathbf{h}_n^{(L)}; \eta^{(L)}, r_k^{(L)}, b^{(L)}\right) , & \mathbf{h}_n^{(L)} &= \mathbf{1}\left(\mathbf{z}_n^{(L+1)}\right) , \end{aligned} \qquad (4)$$

where

- Function $\mathbf{1}(\cdot)$ is defined component-wise as

$$h_{nk}^{(\ell)} = 1 \ \text{ if } \ z_{nk}^{\ell+1} > 0 , \qquad \text{otherwise } \ h_{nk}^{(\ell)} = 0 . \qquad (5)$$

- For top layer

$$z_{kn}^{(L+1)} \sim \text{Poisson}\left(\lambda_k^{(L+1)}\right) , \qquad \lambda_k^{(L+1)} \sim \text{Gamma}\left(a_0, b_0\right) . \qquad (6)$$

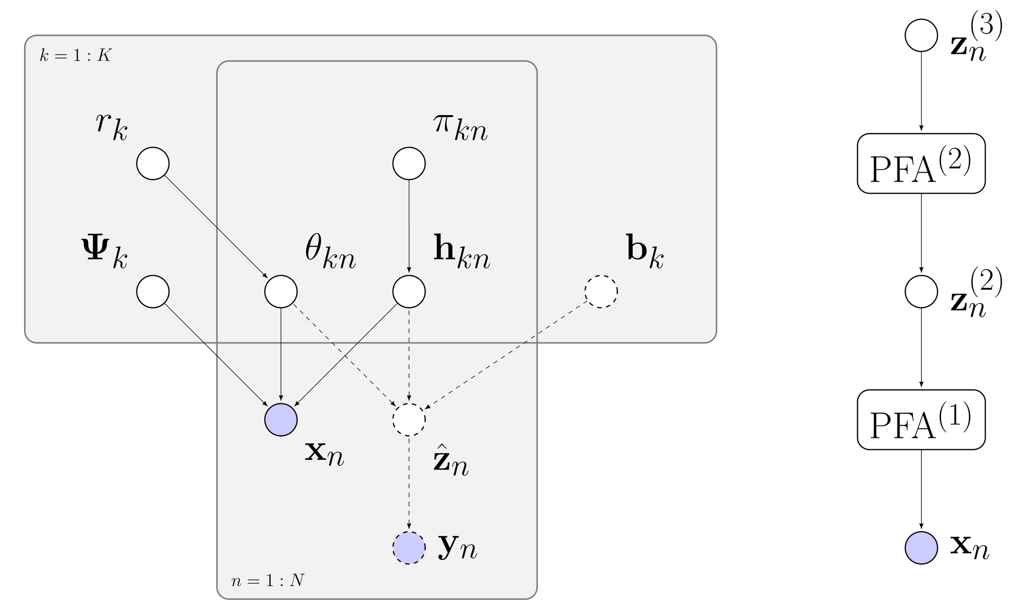
Binary units are constituted as [1]

$$h_{kn}^{(\ell-1)} = 1\left(z_{kn}^{(\ell)} \ge 1\right) , \qquad z_{kn}^{(\ell)} \sim \text{Poisson}\left(\tilde{\lambda}_{kn}^{(\ell)}\right) , \qquad \tilde{\lambda}_{kn}^{(\ell)} = \sum_{k'=1}^{K_\ell} \psi_{kk'}^{(\ell)} \theta_{k'n}^{(\ell)} h_{k'n}^{(\ell)} . \qquad (7)$$

Equivalently

$$p(h_{kn}^{(\ell-1)} = 1) = \text{Bernoulli}\left(\pi_{kn}^{(\ell)}\right) , \qquad \pi_{kn}^{(\ell)} = 1 - \exp\left(-\tilde{\lambda}_{kn}^{(\ell)}\right) . \qquad (8)$$

Inference: Analytic Gibbs updates due to local conjugacy. SVI for large datasets.



(a) (b)

FIGURE 1: Graphical models. (a) PFA module. Nodes ($\mathbf{b}_k$, $\hat{\mathbf{z}}_n$ and $\mathbf{y}_n$) and edges drawn with dashed lines correspond to the discriminative PFA. (b) DPFM.

PFA modules for discriminative tasks

Assume that there is a label $y_n \in \{1,\ldots,C\}$ associated with document n. We impose the model

$$\widehat{\mathbf{y}}_n \sim \text{Multinomial}\left(1, \widehat{\boldsymbol{\lambda}}_n\right) , \qquad \widehat{\lambda}_{cn} = \lambda_{cn} / \textstyle\sum_{c=1}^C \lambda_{cn} , \qquad (9)$$

where

- y_n is represented as a C-dimensional *one-hot* vector, $\widehat{\mathbf{y}}_n$.
- $\boldsymbol{\lambda}_n = \mathbf{B}(\boldsymbol{\theta}_n^{(1)} \circ \mathbf{h}_n^{(1)})$ and λ_{cn} is element c of $\boldsymbol{\lambda}_n$.
- $\mathbf{B} \in \mathbb{R}_+^{C\times K}$, matrix of nonnegative classification weights.
- $\mathbf{b}_k \sim \text{Dirichlet}(\zeta \mathbf{1}_C)$, for $\mathbf{b}_k$ column of $\mathbf{B}$.

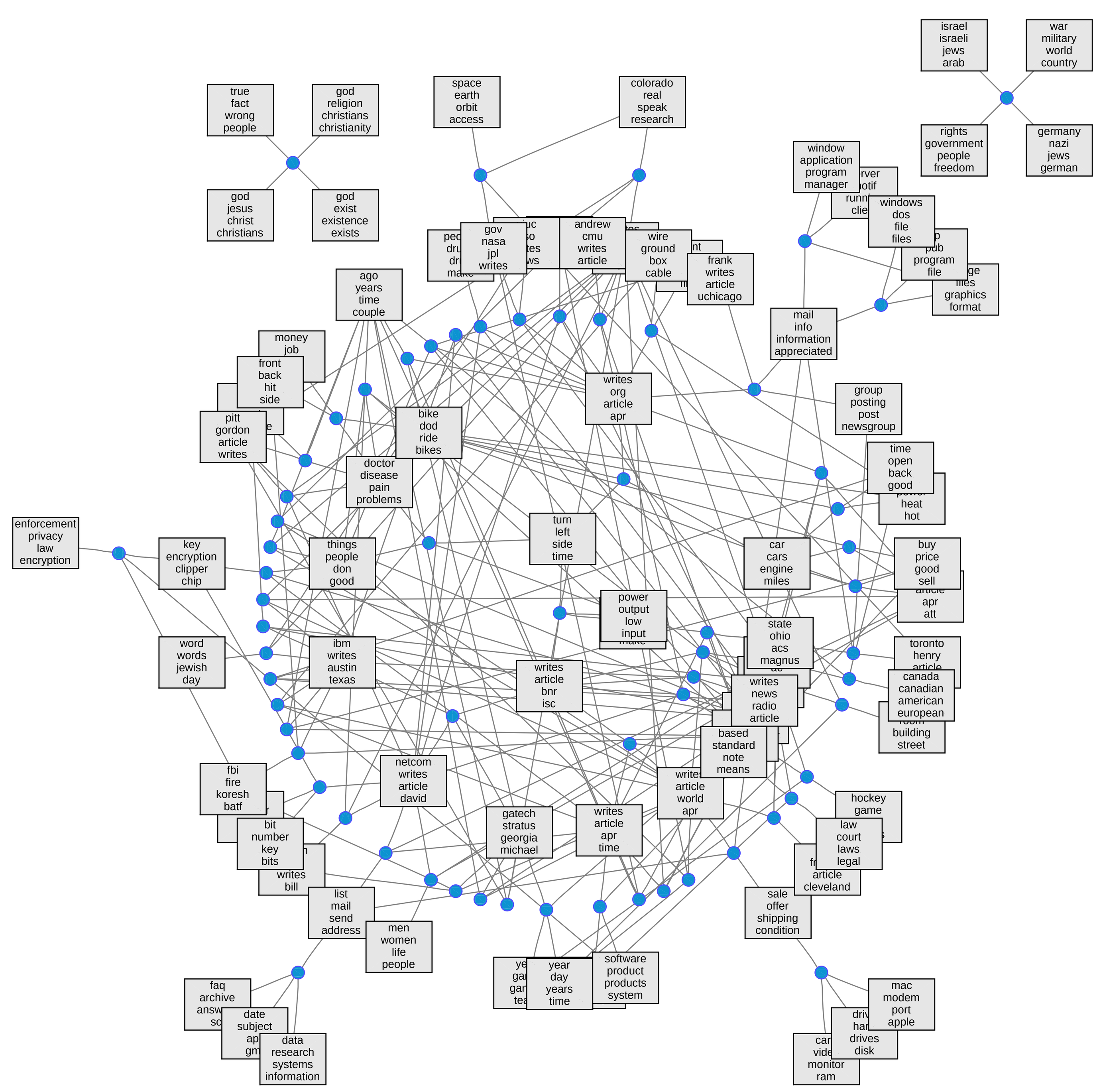


TABLE 1: Graph representation obtained from 20 News. Meta-topics are denoted by circles and layer-1 topics as boxes, with word lists corresponding to the top four words in layer-1 topics, $\boldsymbol{\psi}_k^{(1)}$. We only show the top four connections between meta-topics and their associated topics

Experiments

Benchmark corpora

- Data:
  - 20 Newsgroups (20 News): 2,000 words, 11,315/7,531 training/test documents.
  - Reuters corpus volume I (RCV1): 10,000 words, 794,414/10,000 training/test documents.
  - Wikipedia (Wiki): 7,702 words, 10^7/1,000 training/test documents.
- Performance: held-out perplexity on 20% of test set.
- Models: LDA, FTM, RSM, nHDP, DPFA-SBN, DPFA-RBM and DPFM.
- Runtime: one iteration of the two-layer DPFM on 20 News takes approx. 3/2 secs, for MCMC/SVI.

TABLE 2: Held-out perplexities for 20 News, RCV1 and Wiki. Size: number of topics and/or binary units.

Model	Method	Size	20 News	RCV1	Wiki
DPFM	SVI	128-64	818	961	791
DPFM	MCMC	128-64	**780**	**908**	783
DPFA-SBN	SGNHT	1024-512-256	—	942	**770**
DPFA-SBN	SGNHT	128-64-32	827	1143	876
DPFA-RBM	SGNHT	128-64-32	896	920	942
nHDP	SVI	(10,10,5)	889	1041	932
LDA	Gibbs	128	893	1179	1059
FTM	Gibbs	128	887	1155	991
RSM	CD5	128	877	1171	1001

Classification

- Data: 20 News for document classification.
- Performance: test accuracy.
- Models: LDA, DocNADE, RSM, OSM and DPFM.

TABLE 3: Test accuracy on 20 News. Subscript accompanying model names indicate their size.

Model	LDA_{128}	DocNADE_{512}	RSM_{512}	OSM_{512}	DPFM_{128}	DPFM_{128-64}
Accuracy (%)	65.7	68.4	67.7	69.1	72.11	**72.67**

DPFM also outperforms multinomial logistic regression, SVM, supervised LDA and two-layer feed-forward neural networks, for which test accuracies ranged from 67% to 72.14%, using term frequency-inverse document frequency features.

Medical records

- Duke University 5-year dataset (2007-2011): 240,000 patients, 4.4M visits.
- 34,000 medication mapped to 1,691 pharmaceutical active ingredients (AI).
- Dataset: 1,019×131,264 counts matrix of AIs vs. patients.
- MCMC-based DPFM of size 64-32.

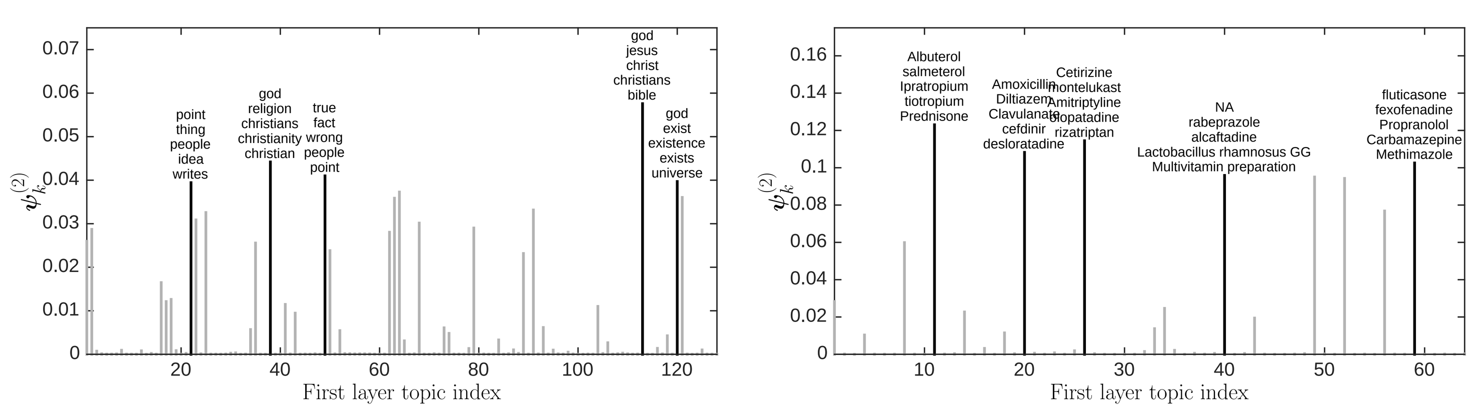


TABLE 4: Representative meta-topics obtained from (left) 20 News and (right) medical records. Meta-topic weights $\boldsymbol{\psi}_k^{(2)}$ vs. layer-1 topics indices, with word lists corresponding to the top five words in layer-1 topics, $\boldsymbol{\psi}_k^{(1)}$.

References

[1] M. Zhou. Infinite edge partition models for overlapping community detection and link prediction. In *AISTATS*, 2015.

[2] M. Zhou, L. Hannah, D. Dunson, and L. Carin. Beta-negative binomial process and Poisson factor analysis. In *AISTATS*, 2012.